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Stochastic Modeling of Compressive Strengths of Some Nigerian Grown Timbers

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STOCHASTIC MODELING OF COMPRESSIVE STRENGTHS OF SOME NIGERIAN GROWN TIMBERS

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ABSTRACT

Natural materials such as timber, stone and many others as used in buildings and other structural systems are laden with uncertainties and often prone to risk of failure due to the inherent variability in their engineering properties. To assess the randomness in properties of typical Nigerian grown timbers and propose stochastic models for used in reliability based design of timber structures, a number of timber species were obtained and assessed in the laboratory. Six timber species with respective local and scientific names of Abura (Mitragyna ciliata), Mahogany (Khaya senegalensis), White-Afara (Terminalia superba), Ire (Futumia ebrifu), Madobiya (Pterocarpus erinaceus) and Ayinre (Albizia lebbek), dried to average moisture content of 7.5% were employed for the study. Simple clear specimens of varied dimensions were prepared and tested under uni-axial compressive strength (oriented in parallel and perpendicular to the timber grains) as well as assessment of density and stiffness (modulus of elasticity) in accordance with BS EN 13183-1 (2002), BS EN 408 (1995) and BS EN 408 (2003) using the Cussons universal testing machine of 100 kN capacity at a loading rate of 0.17 mm/s were carried out. Results from the laboratory analysis were further used in proposing the stochastic model using a Matlab toolbox for statistical and simulation analysis where the best stochastic model (that with the least kolmogorov-Smirnov (K-S) test) is adopted. These, alongside the assessed mode of failure for the timber species reported in the paper would help structural engineers in making informed decisions on the use of these timber materials in design of reliable and safe structures.

Keywords: Timber, Stiffness, Compressive Strength, Stochastic model, Failure modes

INTRODUCTION

Timber applications in building and other structures have been for centuries and to date due to its edge over other materials such as steel, concrete, masonry etc. owing to its renewable resource, light weight and relatively high strength-to-weight ratio (Manfred, 2008; American wood council 2015).

However, use of timber in its natural state require specific assessment of its characteristics after drying due to its varied degree of thermophysical properties and initial moisture content for informed decisions on its suitability for structural application (Diego and Stavros, 2003). Properties such as density, moisture content, fire resistance, resistance to insect attack etc. as well as its engineering properties are normally assessed and stress graded based on correlation between grade determining properties like strength, stiffness, density and measured physical properties.
The properties of well known timbers, broadly divided into softwood (caniferous trees) such as spruce, pine, Douglas fir etc. and hardwood (broad leaved trees) such as oak, ash, mahogany etc. (Seward-1998) are well reported in literature (Taylor and Bender, 1991; Leicester et al., 1996; Isaksson, 1998) except for complex underlying process of ageing and deterioration as well as the anisotropic nature of these wood that are still active subject of research (Jochen 2015; Alberto et al., 2016).

Another challenge associated with the use of timber for structural applications is its variability in properties due to location. Reports have it that timbers from similar trees grown from different region of the world exhibit different properties due to factors such as soil, climate and other environmental conditions (Aguwa, 2015; Hansson and Thelandersson, 2002). Similarly, strength properties of timbers are reported to vary in different parts of its cross section as well as between and within the material (Kohler, 2006; Aguwa, 2015).

There are over 900 species of timbers presently grown in Nigeria (Ihenyen et al. 2009) of which only a few are assessed and explored in structural applications while others used for similar purpose are largely based on empirical judgments.

To this trend, studies on the mechanical properties of some Nigerian grown timbers have been reported. Here the woods: Itako (*strombosia pustulata*), Opepe (*Nauclea diderrichii*) and Ijebu (*drophragma cyclindrcum*) based on EN 338 (2009) limits are classified as hardwood with average bending strengths (parallel to grain) of 64.91 N/mm², 49.89 N/mm², and 44.55 N/mm² respectively (Mohammed and Idris 2016; Deleb palm wood and a host of others have been also assessed to be good for structural applications (Aboshio and Lawal, 2019; Okai, 2008; Oduor and Githiomi, 2009; Kwame, 2001; Kelechi, 2015).

In order to broaden the database on basic properties of Nigerian grown timbers and instill confidence in its structural applications. This study thus, seeks to assess some physical and engineering properties of six timber types, understand their variability and specify statistical distribution type and parameters of six species of timbers for use in reliability analysis and design of timber structures.

**MATERIALS AND METHODS**

**Materials**

*Sample Collection*

Six timber species were used for this study. These are in their local and scientific names: Abura (*Mitragyna ciliata*), Mahogany (*Khaya senegalensis*), White Afara (*Terminalia superba*), Ire (*Futumia ebrifu*), Madobia (*Pterocarpus erinaceus*) and Ayinre (*Albizia labbek*).

**Specimen Preparation**

Logs of the timber species were sawn to clear specimen sizes and end surfaces of the timber polished to ensure that the end surfaces are plane and parallel to one another and perpendicular to the axis of the pieces as specified in BS EN408 (1995).

**Methods**

*Moisture content and density*

The moisture content test was carried out in accordance with EN 408 (2003) and ASTM D193 (2000) standards for specimens of the six samples. The moisture contents (MC) for all the specimens were then computed using Equation 1.

\[
MC = \frac{m_1 - m_2}{m_2} \times 100\% \quad (1)
\]
Where; \( m_1 \) and \( m_2 \) are the initial mass (in grams), final mass (in grams) and MC is the moisture content (in percentage) of the test slice respectively.

The densities of the specimens were determined in accordance with EN 408 (2003) using Equation (2).

\[
\text{Density } (\rho) = \frac{\text{Mass}}{\text{Volume}} \tag{2}
\]

Where mass of the specimens were measured in kilograms, volume measured in cubic meters and density in kilograms per meter cube of the specimen.

**Compressive strength parallel and perpendicular to the grains**

The strength characteristics of forty (40) specimens each of the six timber species were determined using the Cussons' made Universal Testing Machine (UTM) model P5030-100.

Compressive strength parallel and perpendicular to the grain were determined in accordance with BS EN408 (1995). The test pieces were loaded concentrically using a square flat plate loading-heads which permit the application of a compressive load without inducing bending. After an initial load, the loading-head was further used to grip/lock the specimen in-between the two flat plates to prevent angular movement.

The load was applied gradually at a rate of 0.17mm/sec up to failure using the UT machine and the load-extension data were recorded at a frequency of 10 data per second. The compressive strengths of the specimens were computed from Equations 3.

\[
f_{c,0}, f_{c,90} = \frac{F_{\text{max}}}{A} \tag{3}
\]

Where \( F_{\text{max}} \) is the load at failure read directly from the machine, \( A \) is the average area of the bearing surface of the test piece and \( f_{c,0} \) and \( f_{c,90} \) are the Compressive strengths parallel and perpendicular to the grain respectively.

From the data received, stress-strain relations for the respective timbers were computed and their modulus determined from the gradient of plots of the stress against the strain.

**RESULTS AND DISCUSSION**

**Moisture Content (MC)**

The average moisture contents after conditioning of the timber samples are presented in Figure 1. The results from the figure shows that the timbers assessed can be generally adjudged as dry with general average moisture of 7.16%. Specific findings indicate that Ire has the highest average moisture content of 9.22%, followed by White-afara (8.93%), then Ayinre (8.84%), then Mahogany (8.42%), and followed by Abura (7.16%) and Madobiya (6.46%). Noting that the final moisture content is not an exact value but a distribution within a certain range of dispersion, standard deviation of the MC for Abura, Mahogany, White Afara, Ire, Madobiya and Ayinre are 0.42%, 0.49%, 0.75%, 1.56%, 0.59%, and 1.13% respectively. These suggest that the timber samples and less susceptible to decay as with wet timbers (MC > 18%) (Diego and Stavros, 2003).
Figure 1: Average Final Moisture Content for the Six Timber Species

Density

Figure 2 shows that the average density values of the timber species of Abura, Mahogany, White Afara, Ire, Madobiya and Ayinre with corresponding standard deviation in parentheses are 720 (39.19), 1186 (56.33), 806 (22.03), 870 (22.62), 922 (41.51) and 838 (79.30) kg/m³ respectively. These results are consistent with the lower limit for hardwoods species of class D50 based on EN 338 (2009) at the final moisture content less than 18%.

Figure 2: Average densities of the Six Timber Species
Compressive Strength and Elastic Modulus for Sections Parallel to Grain

Compressive strength \( (f_{c,o}) \) and elastic modulus \( (E_{c,o}) \) parallel to grain for varying sections of the timber species considered in this study were obtained alongside their respective standard deviations (ST dv). These are presented in Tables 1.

From the results, it can be seen that Madobiya (when compared with other species) have the highest average compressive strength and modulus of elasticity values of 42.63 and 1376.18 N/mm² for clear specimens 25x25x150 mm. Whereas White Afara was observed to be the lowest in terms of its average compressive strength and modulus of elasticity when compared with Abura. Given also the lower limit for class D50 timber under compressive stress-parallel to grain (32.6 N/mm²) and mean elastic modulus of 1400 N/mm²; it then suggest that Madobiya and Mahogany fit class D50 while Abura, Ayinre and Ire fit class D40 and Afara class D35 of the deciduous specie-D class of hardwoods.

Table 1: Compressive Strength and Elastic Modulus for Varying Sections Parallel to Grain of the Timber Species

<table>
<thead>
<tr>
<th>Size/25 x 25 x 150 mm</th>
<th>Timber Type</th>
<th>( f_{c,o} ) N/mm²</th>
<th>ST dv N/mm²</th>
<th>( f_{c,o} ) N/mm²</th>
<th>ST dv N/mm²</th>
<th>( E_{c,o} ) N/mm²</th>
<th>ST dv N/mm²</th>
<th>( E_{c,o} ) N/mm²</th>
<th>ST dv N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 x 25 x 150 mm</td>
<td>Abura</td>
<td>31.1</td>
<td>4</td>
<td>21.93</td>
<td>8.52</td>
<td>1053.21</td>
<td>464.85</td>
<td>2455.59</td>
<td>1210.15</td>
</tr>
<tr>
<td>25 x 25 x 450 mm</td>
<td>Mahogany</td>
<td>40.84</td>
<td>7.14</td>
<td>34.9</td>
<td>11.42</td>
<td>877.04</td>
<td>210.39</td>
<td>4489.19</td>
<td>1512.71</td>
</tr>
<tr>
<td>25 x 25 x 150 mm</td>
<td>White Afara</td>
<td>24.93</td>
<td>2.87</td>
<td>12.42</td>
<td>1.7</td>
<td>524.25</td>
<td>11.74</td>
<td>1045.32</td>
<td>60.98</td>
</tr>
<tr>
<td>25 x 25 x 450 mm</td>
<td>Ire</td>
<td>26.2</td>
<td>1.83</td>
<td>26.2</td>
<td>8.26</td>
<td>973.17</td>
<td>65.37</td>
<td>2950.09</td>
<td>1202.13</td>
</tr>
<tr>
<td>25 x 25 x 150 mm</td>
<td>Madobiya</td>
<td>42.63</td>
<td>21.08</td>
<td>26.27</td>
<td>9.05</td>
<td>1376.18</td>
<td>79.97</td>
<td>3007.82</td>
<td>935.3</td>
</tr>
<tr>
<td>25 x 25 x 450 mm</td>
<td>Ayinre</td>
<td>30.66</td>
<td>3.63</td>
<td>24.02</td>
<td>1.1</td>
<td>829.37</td>
<td>64.13</td>
<td>3001.43</td>
<td>274.99</td>
</tr>
</tbody>
</table>

The results as also presented in Figure 3 generally show that the compressive capacities of the timber species reduces with increase in the effective lengths of the specimens due to buckling effect. The specimens stiffness however, increases with length as can be seen from Figure 4 for the limit of 450 mm length considered in this study.

Here the compressive strength for Abura falls by 30% (31.10 to 21.93 N/mm²), Mahogany by 15% (from 40.84 to 34.90 N/mm²), White Afara by 50% (from 24.93 to 12.42 N/mm²), Ire by 35% (from 26.20 to 17.14 N/mm²), Madobiya by 38% (from 42.63 to 26.27) and Ayinre by 22% (from 30.66 to 24.02 N/mm²). It is observed that for 25 mm × 25 mm × 450 mm specimen, Mahogany have the highest average compressive strength compared with the other species (Ire, Madobiya and Ayinre).
Figure 3: Compressive Strength Results for Varying Sections of the Timber Species

Figure 4: Elastic Modulus Results for Varying Sections of the Timber Species

Mode of failure of timber species in compression parallel to grain

Three distinct modes of failures were observed for specimens of the timber samples oriented parallel to the timber grains. These are: crushing, shearing and splitting modes of failures as shown in Figures 6. Crushing failure was exhibited by specimens with length 150 mm while shear failure is exhibited by specimens with length 300 mm. In splitting failure, which was specific to White Afara, the specimen divides or splits into two parts as observed in specimens with length 300 mm of the samples. Buckling
effect was also observed in specimens with length 450 mm and 300 mm due to the slenderness of the section as also presented in Figure 6(D).

![Sample images of timber failure modes](image)

Figure 6: Typical shearing (A), crushing (B), splitting (C), and buckling (D) failure modes of timber samples oriented parallel to grain and under compression.

**Compressive strength test perpendicular to grain for the timber species**

Tables 3 and 4 present the results obtained for the compressive strength \( f_{c,90} \) and modulus of elasticity \( E_{c,90} \) perpendicular to grain for the timber species considered in this study.

From results in the tables, the average compressive strength values for 25 x 25 x 150 mm for Abura, Mahogany, White Afara, Ire, Madobiya and Ayinre in compression perpendicular to grain are 30.58, 39.36, 12.07, 18.43, 21.94 and 10.89 N/mm\(^2\) with coefficient of variance of 0.18, 0.70, 0.09, 0.12, 0.23 and 0.12 respectively, and their average modulus of elasticity are 349.61, 395.83, 36.83, 54.63, 73.06 and 50.49 N/mm\(^2\) with coefficient of variance of 0.40, 1.06, 0.07, 0.05, 0.33 and 0.07 respectively.

These results surpasses the lower limit for the D50 class timber and hence the classification based on strength values parallel to grain can be uphold for all the timber species.

**Table 2: Compressive Strength for Varying Sections Perpendicular to Grain of the Timber Species**

<table>
<thead>
<tr>
<th>Size/</th>
<th>25<em>25</em>50 mm</th>
<th>25<em>50</em>50 mm</th>
<th>50<em>50</em>50 mm</th>
<th>50<em>50</em>75 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Timber Type</td>
<td>( f_{c,90} ) N/mm(^2)</td>
<td>ST dv N/mm(^2)</td>
<td>( f_{c,90} ) N/mm(^2)</td>
<td>ST dv N/mm(^2)</td>
</tr>
<tr>
<td>Abura</td>
<td>30.58</td>
<td>5.57</td>
<td>10.18</td>
<td>2.25</td>
</tr>
<tr>
<td>Mahogany</td>
<td>39.36</td>
<td>7.45</td>
<td>29.74</td>
<td>3.4</td>
</tr>
<tr>
<td>White Afara</td>
<td>12.07</td>
<td>1.11</td>
<td>11.86</td>
<td>2.16</td>
</tr>
<tr>
<td>Ire</td>
<td>18.43</td>
<td>2.28</td>
<td>17.55</td>
<td>2.32</td>
</tr>
<tr>
<td>Madobiya</td>
<td>21.94</td>
<td>5.0</td>
<td>22.28</td>
<td>1.22</td>
</tr>
<tr>
<td>Ayinre</td>
<td>10.89</td>
<td>1.3</td>
<td>10.83</td>
<td>0.45</td>
</tr>
</tbody>
</table>
Table 3: Elastic Modulus for Varying Sections Perpendicular to Grain of the Timber Species

<table>
<thead>
<tr>
<th>Size/ Timber Type</th>
<th>25<em>25</em>50 mm</th>
<th>25<em>50</em>50 mm</th>
<th>50<em>50</em>50 mm</th>
<th>50<em>50</em>75 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_{c,90}$ N/mm$^2$</td>
<td>ST dv N/mm$^2$</td>
<td>$E_{c,90}$ N/mm$^2$</td>
<td>ST dv N/mm$^2$</td>
</tr>
<tr>
<td>Abura</td>
<td>960</td>
<td>4.7</td>
<td>937.25</td>
<td>5.23</td>
</tr>
<tr>
<td>Mahogany</td>
<td>1950.83</td>
<td>3.4</td>
<td>1534.81</td>
<td>11</td>
</tr>
<tr>
<td>White Afara</td>
<td>936.83</td>
<td>2.56</td>
<td>929.1</td>
<td>5.53</td>
</tr>
<tr>
<td>Ire</td>
<td>954.63</td>
<td>2.69</td>
<td>957.44</td>
<td>8.42</td>
</tr>
<tr>
<td>Madobiya</td>
<td>1073.06</td>
<td>4.3</td>
<td>982.98</td>
<td>6.2</td>
</tr>
<tr>
<td>Ayinre</td>
<td>950.49</td>
<td>3.68</td>
<td>851.56</td>
<td>7.23</td>
</tr>
</tbody>
</table>

Figure 6 also shows average compressive strength, perpendicular to grain for varying cross section of the specimens considered. Here again the buckling effect is evident for all the specimens except section from madobiya samples. Similar trend was observed for results of the elastic modulus where slight increase were observed for madobiya specimens in contrast with results obtained when the specimens were oriented parallel to the timber grain. The results presented in Figure 6 also show that mahogany exhibit higher stiffness in compression when compared with the other specimens considered.

![Figure 6: Compressive Strength for Varying Section of the Timber Samples](image-url)
Mode of failure of timber species in compression perpendicular to grain

The modes of failure observed in the compressive strength test perpendicular to grain are crushing and shearing failure as shown in Figure 7. Crushing failure is exhibited by specimen with length 50 mm while shear failure is exhibited by specimen with length 75 mm for all specimens of varying cross section of the samples considered.

Figure 7: Elastic Modulus for Varying Section of the Timber Samples

Figure 7: Typical crushing (A) and shearing (B) failure modes of timber samples under compression force perpendicular to grain.
STOCHASTIC MODELLING OF THE PROPERTIES OF THE TIMBER SPECIMENS

Kolmogorov - Smirnov (K-S) Test Results

Table 4 presents the summary of K-S test results for the six selected timber species. The hypothesis regarding the distribution form is rejected if the K-S statistics, D, is greater than the critical value at 0.05 confidence level (i.e. α 0.05). The K-S statistics for the basic material properties considered were all found to be less than the critical values presented in Table 4. As such, any of the four types of distributions considered (Normal, Lognormal, Gumbel and Weibull) can be used to model the timber properties. However, the best distribution type (stochastic model) is obtained by ranking the selected distributions and it corresponded to the one having the least K-S statistic.

Table 4 Kolmogorov Smirnov (K-S) Test Results

<table>
<thead>
<tr>
<th>Timber Property</th>
<th>Density</th>
<th></th>
<th>MOE</th>
<th></th>
<th>Compressive Strength</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Timber</td>
<td>Species</td>
<td>Distribution</td>
<td>Type</td>
<td>Normal</td>
<td>Lognormal</td>
</tr>
<tr>
<td></td>
<td>Abura</td>
<td>D</td>
<td>0.252</td>
<td>0.252</td>
<td>0.28</td>
<td>0.262</td>
</tr>
<tr>
<td></td>
<td>α 0.05</td>
<td>0.338</td>
<td>0.338</td>
<td>0.338</td>
<td>0.338</td>
<td>0.294</td>
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<td>1</td>
<td>4</td>
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<td>3</td>
</tr>
<tr>
<td></td>
<td>Mahogany</td>
<td>D</td>
<td>0.255</td>
<td>0.25</td>
<td>0.28</td>
<td>0.261</td>
</tr>
<tr>
<td></td>
<td>α 0.05</td>
<td>0.338</td>
<td>0.338</td>
<td>0.338</td>
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<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>White-Afara</td>
<td>D</td>
<td>0.252</td>
<td>0.253</td>
<td>0.28</td>
<td>0.262</td>
</tr>
<tr>
<td></td>
<td>α 0.05</td>
<td>0.338</td>
<td>0.338</td>
<td>0.338</td>
<td>0.338</td>
<td>0.294</td>
</tr>
<tr>
<td></td>
<td>Rank</td>
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<td>2</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Ire</td>
<td>D</td>
<td>0.337</td>
<td>0.329</td>
<td>0.274</td>
<td>0.238</td>
</tr>
<tr>
<td></td>
<td>α 0.05</td>
<td>0.338</td>
<td>0.338</td>
<td>0.338</td>
<td>0.338</td>
<td>0.294</td>
</tr>
<tr>
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<td>Rank</td>
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<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Madobiya</td>
<td>D</td>
<td>0.252</td>
<td>0.25</td>
<td>0.28</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>α 0.05</td>
<td>0.338</td>
<td>0.338</td>
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<td>4</td>
<td>3</td>
<td>3</td>
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<tr>
<td></td>
<td>Ayinre</td>
<td>D</td>
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<td>0.222</td>
<td>0.237</td>
<td>0.236</td>
</tr>
<tr>
<td></td>
<td>α 0.05</td>
<td>0.338</td>
<td>0.338</td>
<td>0.338</td>
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<td>1</td>
<td>4</td>
<td>3</td>
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</tr>
</tbody>
</table>
CONCLUSIONS

In this study, stochastic modeling of basic properties of six Nigerian grown timbers namely: Abura (*Mitragyna ciliata*), Mahogany (*Khaya senegalensis*), White-Afara (*Terminalia superba*), Ire (*Futumia ebrifu*), Madobiya (*Pterocarpus erinaceus*) and Ayinre (*Albizia lebbek*) were assessed and key findings from the study are as follows:

1. That at an average moisture content of 7.5 per cent, mean densities for Abura, Mahogany, White Afara, Ire, Madobiya and Ayinre were respective determined as 720, 1186, 806, 870, 922, 838 kg/m$^3$fitting to densities of hardwoods in class D as presented in EN 338 (2009).

2. Given the lower limit for class D50 timber under compressive stress-parallel to grain (32.6 N/mm$^2$) and mean elastic modulus of 1400 N/mm$^2$; it then suggest that Madobiya and Mahogany fit class D50 while Abura, Ayinre and Ire fit class D40 and Afara class D35 of the deciduous specie-D class of hardwoods.

3. Compressive strength values for all specimens considered in this study decreases with increase in the specimen length except for Ayinre where length of the sample have little effect on its compressive strength.

4. The compression failure patterns obtained from this study were similar to those defined in ASTM D143. Crushing and shearing failure pattern were the most common mode of failure of the timber species in compression both parallel and perpendicular to grain.

5. Stochastic modeling and statistical distribution of parameters of density, compressive strength and stiffness of the six timber samples suggest that the parameters can respectively best be modeled using lognormal, weibull and gumbel distributions.

REFERENCES


